# A Revised Model for Fuzzy Multi Choice Goal Programming

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**Abstract.** Goal programming (GP) is one of the most common and fundemental approaches to solve Multi Objective Decision Making (MODM) problems. However, in some cases in real world situation, there are more than just one aspiration level for each objective function. Recently, some authors have been introduced Multi Choice Goal Programming (MCGP) models to solve such problems. Then, some other studies have been given for the applicability of this concept in our daily problems. One of these comfortable models is concerning to Fuzzy Multi Choice Goal Programming (FMCGP) which is more adapt to the real situations. Hence, in this paper we concentrate on these models and in particular propose a new approach to solve FMCGP problems. Moreover, a numerical discussion has given to show the sutability and efficiency of the suggested model. We see that the proposed model has some advantages in comparison of the common models which are well-known in the literature.

**Keywords:** Goal programming · Multi-choice goal programming · Fuzzy mathematical programming · Fuzzy goal programming

## 1 Introduction

Linear Programming (LP) is one of the fundemental concepts which is heavily used in real life problems such as planning, production, transportation and technology. So finding some approaches to make the model appropriate for real world situations, is really important. Since Goal Programming (GP) was first recommended by Charnes et al. [9], it has been one of the most widely used techniques to solve Multi Objective Decision Making (MODM) problems. It was first suggested by Charnes et al. [9], and latter develped by Lee [13], Ignizio [11], Tamiz et al. [21] and Romero [19]. GP is used to reduce the unwanted deviation of goals values and their Aspiration Levels (ALs) in order to find a set of satisfying solutions.

Sometimes in our daily lives problems, the Decision Maker (DM) would like to make some different Als for each target. It is not possible to solve These problems by the classic GP approaches. To deal with such problems, in 2007 Chang proposed a Multi


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Choice Goal Proramming (MCGP) approach which allows the DM to set a Multi Choice Aspiration Levels (MCAL) for each objective [4]. The first typical MCGP problems was introduced as follows:

$$\min \sum\nolimits_{i=1}^{n} |f_i(x) - h_{i1} \text{ or } h_{i2} \text{ or } \ldots \text{ or } h_{im}| \tag{1}$$

$$\text{s.t.} \quad x \in F \quad (F \quad \text{is a feasible set}) \tag{2}$$

where $h_{ij}$ $(i = 1, 2, \ldots, n\, and\, j = 1, 2, \ldots, m)$ is the jth AL of the ith goal and the achievement function was proposed as:

$$\min \sum_{i=1}^{n} w_i\left(d_i^+ + d_i^-\right) \tag{3}$$

$$\text{s.t.} \quad f_i(x) - d_i^+ + d_i^- = \sum_{j=i}^{m} h_{ij} S_{ij}(B), \qquad i = 1, 2, \ldots, n \tag{4}$$

$$d_i^+, d_i^- \geq 0, \; i = 1, 2, \ldots, n, \tag{5}$$

$$S_{ij}(B) \in R_i(x), \; i = 1, 2, \ldots, n, \tag{6}$$

$$x \in F \quad (F \quad \text{is a feasible set}) \tag{7}$$

where $S_{ij}(B)$ illustrates a function of binary variables; $R_i(x)$ is a function of resources limitations. Using binary variables specially while the given problem has a large size, may makes it hard to understand. So later in 2008, Chang proposed a new MCGP model without using any binary variables [5]. In this model Chang employed a continuos variable, $y_i$, $\left(h_{i,\min} \leq y_i \leq h_{i,\max}\right)$ with a range of interval values, to exchange multiplicative terms of binary variables. So for instance for the case of the more is the better, the revused MCGP-achievement can be formulated as follows:

$$\min \sum\nolimits_{i=1}^{n} \left[w_i\left(d_i^+ + d_i^-\right) + \alpha_i\left(e_i^+ + e_i^-\right)\right] \tag{8}$$

$$\text{s.t.} \quad f_i(x) - d_i^+ + d_i^- = y_i, \qquad i = 1, 2, \ldots, n \tag{9}$$

$$y_i - e_i^+ + e_i^- = h_{i,\max}, \qquad i = 1, 2, \ldots, n \tag{10}$$

$$h_{i,\min} \leq y_i \leq h_{i,\max} \tag{11}$$

$$d_i^+, d_i^-, e_i^+, e_i^- \geq 0, \qquad i = 1, 2, \ldots, n \tag{12}$$

where $d_i^+, d_i^-$ are the pos/ neg deviations related to $\left|y_i - h_{i,\max}\right|$; $\alpha_i$ is the weight related to the sum of the deviations of $\left|y_i - h_{i,\max}\right|$; other variables are determined as in GP-achievement.

Later Biswal and Acharya solved multi choice linear programming problems by interpolating polynomials [3], Chang also proposed a new approach to solve MCGP models with utility function [6]. Due to the applicability and efficiency of MCGP, it has been gained attention of many researchers [1, 7, 8, 10, 12, 14–16, 18, 22]. In 2012, Tabrizi et al. formulated the Fuzzy Multi Choice Goal Programming (FMCGP) problems with the use of fuzzy membership function and multiplicative binary variables to make it more efficient to work on various real world problems [2, 23–28]. However, the number of constraints in an MCGP problem specially when the size gets large leads to difficult implementation, in this paper a novel model for solving FMCGP problems is suggested. One interest thing about this model is it involves less constraints and at the same time leads to a better solution.

The rest of the paper is organized as follows: In Sect. 2, the Fuzzy MCGP is demonstrated. Further in Sect. 3, a new approach to solve FMCGP problems without the use of membership function is proposed. Moreover due to indicate the efficiency of the suggested model a comparitive example is shown in Sect. 4 and also a comparison is made among the presented model and an existing model. Eventually, conclusions are explained in Sect. 5.

## 2 Fuzzy MCGP

In some real world decision making situations, the exact and crisp values for each AL cannot be determined. In 2012, for the first time in history, Tabrizi et al. formulated an FMCGP approach to solve such problems. The above mentioned FMCGP is formulated as follows [2]:

$$\min \sum_{n=1}^{i} w_i \left| f_i(x) - \widetilde{h_{i1}} \text{ or } \widetilde{h_{i2}} \text{ or } \ldots \text{ or } \widetilde{h_{im}} \right| \tag{13}$$

$$\text{s.t.} \quad x \in F \quad (F \quad \text{is a feasible set}) \tag{14}$$

where $w_i, i = 1, 2, \ldots, n$ is the weight for the ith goal; $h_{ij}, j = 1, 2, \ldots, m$ is the jth AL of the ith objective function which is a triangular fuzzy number with membership function $\mu_i$ [17]. Now by using the fuzzy sets theory and multiplicative binary variables, the current FMCGP model can be represented as follows:

$$\max \quad f(\mu) = \sum_{n=1}^{i} w_i \mu_i \tag{15}$$

$$\text{s.t.} \quad \mu_i \leq 1 - \sum\nolimits_{j=1}^{n} \frac{H_i(x) - \widetilde{h_{ij}}}{d_{ij}^{-}} S_{ij}(B), \qquad i = 1, 2, \ldots, n, \tag{16}$$

$$\mu_i \leq 1 - \sum\nolimits_{j=1}^{n} \frac{\widetilde{h_{ij}} - H_i(x)}{d_{ij}^{+}} S_{ij}(B), \qquad i = 1, 2, \ldots, n \tag{17}$$

$$x \in F \quad (F \quad \text{is a feasible set}) \tag{18}$$

$$\mu_i \geq 0, \quad i = 1, 2, \ldots n, \tag{19}$$

where $S_{ij}(B)$ represents a function of binary variables; $d_{ij}^-$, $d_{ij}^+$ are the maximum admissible neg/pos deviations from the jth AL in the ith target; $H_k(x)$ denotes the kth objective function.

# 3 The Proposed Model

Most real world decision making problems have a large number of constraints which makes the problem a complex problem and cause a waste of time, energy and etc. Therefore in this section, a novel approach is presented to solve FMCGP problems.

**Definition 3.1.** We define the Weighted Goal Programming (WGP) as follows:

$$\min \quad \sum_{i=1}^{n} w_k \left( \sum\nolimits_j S\left( \frac{d_i^+}{\Delta_{ij}^+} + \frac{d_i^-}{\Delta_{ij}^-} \right) S_{ij}(B) \right) \tag{20}$$

$$\text{s.t. } H_i(x) - d_i^+ + d_i^- = \widetilde{h_{ij}} S_{ij}(B), \; i = 1, 2, \ldots, n \tag{21}$$

$$x \in F \quad (F \quad \text{is a feasible set}), \tag{22}$$

$$d_i^+, d_i^- \geq 0, \qquad i = 1, 2, \ldots, n \tag{23}$$

where $\Delta_{ij}^+$ and $\Delta_{ij}^-$ are the maximum admissible violations from the jth AL in the ith goal while $d_i^+$ and $d_i^-$ represent the positive and negative deviations from this AL; $H_i(x)$ denotes the ith objective function and also binary variables are shown with $S_{ij}(B)$.

**Remark 3.1.** The proposed model contains fewer constraints and also leads to a better solution which makes it more useful especially when the size of the problem gets large.

**Remark 3.2.** The suggested model is a linear form of FMCGP that can easily be solved as well as the common linear programming method.

Because of the above main properties which are emphasized as Remark 3.1 and 3.2, we give the main steps of the solution process as below.

## 3.1 Algorithm of the Proposed Model

**Step1:** Transfer the given FMCGP problem into the model (20) which is presented in this section.

**Step2:** Define weights for each objective function.

**Step3:** Obtain the optimal solution of the main problem by solving WGP problem which is defined in model (20).

**Remark 3.3.** The desirable weights in step 2 will be determined based on the point of view of some expert persons.

Now we are at a place to illustrate our suggested approach. Therefore, a numerical example is presented in the following section. Besides this model has some advantages which are investigated in the next section.

## 4 Comparative Study

In this section, an illustrative example from [2] with some changes in it's numbers is used to indicate the performance of the presented model. A factory is producing three products $y_1, y_2$ and $y_3$. For product $y_1$, there are some consumers $A_1$, $A_2$ and $A_3$ with "approximate" demands 40, 60 and 100. The maximum allowable positive and negative deviation of consumers $A_1$, $A_2$ and $A_3$ from their targets are equal and set as 4, 5, 6, accordingly. The advantage of the sale of this good is 10$. Other details related to these goods are represented in Table 1. Moreover, due to some restrictions including poilitical ones the factory has to choose just one of its consumers for each good. The minimum expected profit from goods selling is 850$. Three resources $s_1$, $s_2$ and $s_3$ are required to construct these goods. The amounts of each resource which is required to generate each good are represented in Table 2.

**Table 1.** The range of data relevant to products.

| Products | Customer | Demands | Maximum admissible pos. and neg. deviation | Profit |
|---|---|---|---|---|
| $y_1$ | $A_1$ | 40 | 4 | 10 |
| | $A_2$ | 60 | 5 | |
| | $A_3$ | 100 | 6 | |
| $y_2$ | $B_1$ | 10 | 2 | 12 |
| | $B_2$ | 20 | 3 | |
| $y_3$ | $C_1$ | 30 | 3 | 15 |
| | $C_2$ | 50 | 4 | |

**Table 2.** The amount of consumptions of resources for each product.

| Resources | Product | | |
|---|---|---|---|
| | $y_1$ | $y_2$ | $y_3$ |
| $s_1$ | 5 | 7 | 4 |
| $s_2$ | 3 | 5 | 6 |
| $s_3$ | 1 | 2 | 1 |

The associated mathematical model for the above problem can be formulated as follows:

$$\min \mathrm{w}_1 \left| \mathrm{y}_1 - \widetilde{40} \text{ or } \widetilde{60} \text{ or } \widetilde{100} \right| + \mathrm{w}_2 \left| \mathrm{y}_2 - \widetilde{10} \text{ or } \widetilde{20} \right| + \mathrm{w}_3 \left| \mathrm{y}_3 - \widetilde{30} \text{ or } \widetilde{50} \right| \tag{24}$$

$$\text{s.t.} \quad \mathrm{a}_1 : 10\mathrm{y}_1 + 12\mathrm{y}_2 + 15\mathrm{y}_3 \geq 850, \tag{25}$$

$$a_2 : y_1 \leq \frac{x_{11}}{5}, \tag{26}$$

$$a_3 : y_1 \leq \frac{x_{12}}{3}, \tag{27}$$

$$a_4 : y_1 \leq x_{13}, \tag{28}$$

$$a_5 : y_2 \leq \frac{x_{21}}{7}, \tag{29}$$

$$a_6 : y_2 \leq \frac{x_{22}}{5}, \tag{30}$$

$$a_7 : y_2 \leq \frac{x_{23}}{2}, \tag{31}$$

$$a_8 : y_3 \leq \frac{x_{31}}{4}, \tag{32}$$

$$a_9 : y_3 \leq \frac{x_{32}}{6}, \tag{33}$$

$$a_{10} : y_3 \leq x_{33}, \tag{34}$$

$$a_{11} : x_{11} + x_{12} + x_{13} \leq 400, \tag{35}$$

$$a_{12} : x_{21} + x_{22} + x_{23} \leq 380, \tag{36}$$

$$a_{13} : x_{31} + x_{32} + x_{33} \leq 120, \tag{37}$$

Now let us to consider $w_1$, $w_1$ and $w_3$ as 0.4, 0.3 and 0.3, respectively. Therefore, according to the presented approach, the above problem can be reformulated as below:

$$\begin{aligned} \min 0.4\Big(\big(\tfrac{p_1}{4} + \tfrac{q_1}{4}\big)z_1 z_2 + \big(\tfrac{p_1}{5} + \tfrac{q_1}{5}\big)z_1(1 - z_2) + \big(\tfrac{p_1}{6} + \tfrac{q_1}{6}\big)z_2(1 - z_1)\Big) \\ +0.3\Big(\big(\tfrac{p_2}{2} + \tfrac{q_2}{2}\big)z_3 + \big(\tfrac{p_2}{3} + \tfrac{q_2}{3}\big)(1 - z_3)\Big) \\ +0.3\Big(\big(\tfrac{p_3}{3} + \tfrac{q_3}{3}\big)z_4 + \big(\tfrac{p_3}{4} + \tfrac{q_3}{4}\big)(1 - z4)\Big) \end{aligned} \tag{38}$$

$$\text{s.t. } y_1 - p_1 + q_1 = 40z_1z_2 + 60z_1(1 - z_2) + 100z_2(1 - z_1), \tag{39}$$

$$y_2 - p_2 + q_2 = 10z_3 + 20(1 - z_3), \tag{40}$$

$$y_3 - p_3 + q_3 = 30z_4 + 50(1 - z_4), \tag{41}$$

$$z_1 + z_2 \geq 1, \tag{42}$$

**Table 3.** The achieved results from WGP model and Tabrizi's approach.

| | $y_1$ | $y_2$ | $y_3$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $\mu_1$ | $\mu_2$ | $\mu_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Tabrizi's method | 40 | 20 | 10 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| The proposed model | 40 | 10 | 10.9 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |

$$a_i,\ i = 1, \ldots 13. \tag{43}$$

The current problem is solved by software of LINGO [20] and then the obtained optimal solution is $(y_1, y_2, y_3, z_1, z_2, z_3, z_4) = (40, 20, 10.9, 1, 1, 0, 1)$. Moreover, the mentioned problem is also solved by Eq. (15) with LINGO and the optimal solution is obtained as $(y_1, y_2, y_3, z_1, z_2, z_3, z_4) = (40, 20, 10, 1, 1, 1, 1)$. The results are presented in Table 3.

It is clear that the values obtained for the first objective function from Eq. (15) and Eq. (20), both have the same fuzzy achievement degrees that is $\mu_1 = 1$, while for the other objective functions, a better answer is obtained from Eq. (20). Besides, Eq. (20) has fewer constraints, which makes it more practical and also easier to solve.

## 5 Conclusion

This paper developed a new model to solve FMCGP problems by using the multiplicative binary variables. To emphasise the advantages of the new model we made a comparative study with an existing method as a convenient approach which is appeared in the literature. The new model which is a linear form of FMCGP problems, has some advantages including it involves fewer constraints and at the same time leads to a better solution. Besides, as it mentioned once, solving a model with the multiplicative terms of binary variables is difficult to apply while the given problem has a large size. Thus applying a similar approach to solve such problems without any binary variables can be considered as a further investigation.

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