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Differential Search Algorithm based Design of Fractional Order PID Controller for Hard Disk Drive Read/Write System

Anguluri Rajasekhar, *Student Member IEEE*
Department of Electrical Engineering
National Institute of Technology, Warangal
Andhra Pradesh-506004, India
Email: rajasekhar.anguluri@ieee.org

Millie Pant
Department of Paper and Pulp Technology
Indian Institute of Technology-Roorkee
Roorkee-247667, India
Email: millifpt@iit.ernet.in

[1,2]Ajith Abraham, Senior *Member IEEE*
[1]IT4Innovations, VSB-Technical University of Ostrava, Ostrava, Czech Republic
[2]Machine Intelligence Research Labs (MIR Labs), WA, USA
Email: ajith.abraham@ieee.org

***Abstract*—This paper suggests a novel intelligent closed loop control strategy based on fractional order (FO) PID controller for head positioning servo control system. The design of FOPID controller has been formulated as a single objective optimization framework using time domain optimality criterion and is carried out with help of Differential Search (DS) algorithm. In order to digitally realize FOPID controller, an Oustaloup 5th order approximation has been used. It is shown that the servo system optimally moves the reader head on to the desired track when actuated with FOPID controller than a normal PID controller. Validation results of DS algorithm tuned FOPID controller are compared with PID controller, which shows the superior closed-loop response and robustness of the proposed approach.**

Keywords-Disk drive; Intelligent Control; Global optimization; Swarm Intelligence.

I. INTRODUCTION

In most of the computers, magnetic hard disk drives (HDDs) are primary permanent storage devices [1]. Typically in HDDs, data is written on a rotating disk coated with a thin magnetic layer or recording medium. This Data is arranged in concentric circles or tracks. Data is read or written by a radially traversing read/write head that consists of a small horseshoe-shaped magnet. A voice coil motor (VCM) drives the read/write heads to reach the specific track. The aim of head positioning servo mechanism is to position the Read/Write head over desired track with minimum time using a bounded control effort (*Track Seeking*) as well as to maintain the head as close as possible to the destination track center while information is being read from or written onto the disk (*Track Following*).

Currently many hard disk drive servo systems are designed using conventional Proportional-Integral-Derivative (PID) controllers in the track following mode. Proportional-Integral-Derivative (PID) controllers are highly preferred in Industries over several other control algorithms, because of their wide range of operating conditions followed by their functional simplicity and ease in implementation. However, higher demands of storage capacity require more precise positioning of Read/Write heads due to decrease in the width of track. Recent past had witnessed number of modern control approaches for designing controllers that satisfies the current industry requirements.

Hanslemann and Engelke [2] applied continuous time optimal LQG/LTR technique to design a tracking controller for high resonant disk drive actuator, on other hand Weerasooriya and Phan [3] presented the discrete-time version of LQG/LTR design. Goh *et al.* [4] proposed the robust and perfect tracking (RPT) method for servo control system. Hirata *et al.* [5] proposed a new design method for head position controller using multi-rate sampled-data H_∞ control theory. Tan *et al.* [1] applied the concept of multi-objective optimization and developed two-degree-freedom-of controller.

In this paper we propose a Fractional Order PID controller for the Read/Write head position servo mechanism. A FOPID controller ($PI^\lambda D^\mu$ controller) is an extension to conventional PID controller which involves non-integer powers in the integral and derivative operators. FOPID controllers are based on concept of fractional order integrals & derivatives. Recent literature shows that these controllers outperform normal PID controllers if they are designed effectively [6]. Typically, a FOPID controller consists of proportional gain (K_P), Integral gain (K_i) and derivative gain (K_d) constants, order of derivative (μ) and order of Integral (λ). To extract better performances from this controller these 5 constants are to be set optimally. Hence, designing an optimum FOPID controller requires fine tuning of parametric gains which in turn calls for real parameter optimization in five-dimensional hyperspace. To carry out this optimization task we used Differential Search (DS) [7], a recently proposed algorithm based on swarm

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intelligence. The design method focuses on minimization of time domain based optimality criterion. In parallel we also designed optimal PID controller and analyzed both PID and FOPID controllers in terms of time domain indices and also in terms of frequency domain stability.

The rest of paper is organized as follows. Section 2 deals with problem definition followed by fractional order controller introduction and its digital realization in Section 3. A basic introduction of Differential Search (DS) algorithm is presented in Section 4. Experimental studies carried out are shown in Section 5. Conclusions and future work details are disclosed in Section 6.

II. PROBLEM FORMULATION

The control goals to be established in Read/Write head positioning servomechanism are [8, 9]

i. To position the reader head accurately at desired track.

ii. To move reader head from one track to another within minimum time.

An initial system configuration is shown in Figure 1. This system primarily uses a motor to actuate the arm to the desired location on this disk. Here $Head_{Ref}$ refers to desired head position where $Head_{Act}$ corresponds to Actuator head actual position.

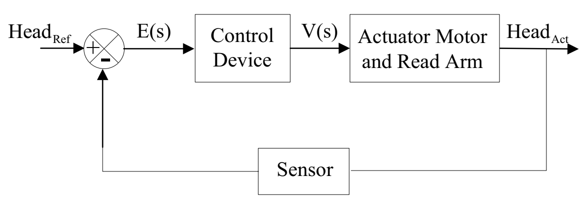

Figure 1. Closed-loop control for HDD Servo System

A. Hard Disk Read/Write System

The Hard Disk Drive reader uses a Voice Coil Motor (VCM) to rotate the reader arm. In industries a Permanent Magnet DC motor [8] is used as VCM. A slider device is connected to the arm on which reader head is mounted. A metallic spring (flexure) helps the head to float above the disk at a gap even less than 0.1μm. The thin film head reads the magnetic flux and then provides signal to control device. Assuming read head to be accurate, the sensor transfer function can be considered as H(s)=1. Parameters of the Hard disk drive system considered for current study are shown in Table 1. The linear transfer function of Voice Coil Motor is given by following Equation 2.1.

$$G_1(s) = \frac{K_m}{Ls + R} \tag{2.1}$$

The transfer function model of load to be driven by voice coil motor is given by

$$G_2 = \frac{1}{s}\left(\frac{1}{Js + b}\right) \tag{2.2}$$

The Hard disk drive must accurately position the reader head and also should reduce the effect of parameter changes, vibrations and external shocks. Disturbances such as physical block, wobble in spindle bearings, component changes will affect the performance of drive. Hence performance of disk drive should be examined in the presences of disturbances. The complete control system for disk drive head reader is depicted in Figure 2.

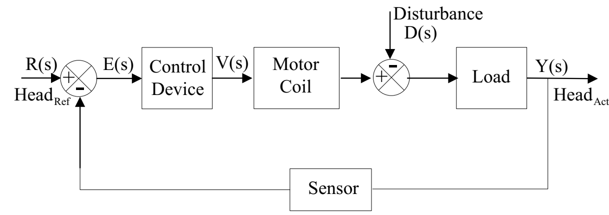

Figure 2. Control System for Hard Disk Drive Read Header

The overall open loop and closed loop transfer functions of disk drive model (when D(s)=0 and error signal is only the control action) are given in Equations (2.4), (2.5)

$$G_{ol} = G_1 \times G_2$$

$$G_{ol} = \left(\frac{K_m}{Ls + R}\right)\cdot\frac{1}{s}\cdot\left(\frac{1}{Js + b}\right) \tag{2.3}$$

$$G_{ol} = \frac{5000}{(s+1000)\cdot s\cdot(s+20)} = \frac{5000}{s^3 + 1020s^2 + 20000s} \tag{2.4}$$

$$G_{cl} = \frac{G_{ol}(s)}{1 + G_{ol}(s)\cdot H(s)} = \frac{5000}{s^3 + 1020s^2 + 20000s + 5000} \tag{2.5}$$

Table 1. Parameters for disk drive

Symbol	Parameter	Value
K_m	Motor Constant	5 N-m/A
L	Armature Inductance	1 mH
R	Armature Resistance	1 Ω
b	Friction	20 kg/m/s
J	Inertia of arm and rotor	1 N-m-sec^2/rad

B. Design of PID/FOPID Controllers

To satisfy the control goals of Read/Write system, suitable control structures are to be designed. Hence to achieve this task we suggest an intelligently tuned PID and FOPID controllers. PID controller is a weighted sum of integration and derivative operators & its transfer function formulation is given by Eq (2.6). On the other hand a FOPID controller is also a weighted sum of integral and derivative operators but of fractional nature and its transfer function is given by Eq (2.7). Discussion on FOPID controller is deferred until section III.

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d s \tag{2.6}$$

$$G_{FOPID}(s) = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \tag{2.7}$$

The main goal is to obtain the unknown parameters of these controllers to achieve optimal performance of drive considered. To carry out this task an optimization frame work based on error signal is developed and is performed using DS algorithm.

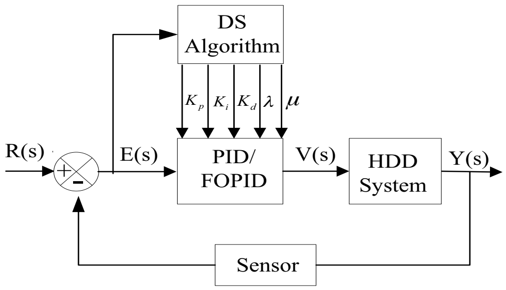

Figure 3. Block Diagram representation of tuning Controllers

Fig 3 shows the block diagram representation of tuning controllers with respect to error signal. Objective functions based on time domain optimality criterion are considered. Following subsection provides further information.

C. Objective Function

PID/FOPID controller parameters are tuned in such way that the drive gives the required performance. For tuning of controllers, we considered three objective functions viz. Integral Absolute Error (IAE), Integral Time Squared Error (ITSE) and Integral Time Absolute Error (ITAE) criterion. The optimal parameters of PID/FOPID controller are obtained by minimizing these objective functions. Equations (2.8 - 2.10) provides the mathematical representation of these objective functions.

$$J_1 = IAE = \int_0^\infty |e(t)| dt \qquad (2.8)$$

$$J_2 = ITSE = \int_0^\infty t \cdot e^2(t) dt \qquad (2.9)$$

$$J_3 = ITAE = \int_0^\infty t \cdot |e(t)| dt \qquad (2.10)$$

Every time domain optimality criterion has its own advantage in designing control systems. ITSE has an advantage of damping out oscillations quickly due to the presence of time multiplication term. While ITAE term penalizes error more at later stages and helps in reducing time indices like settling time and overshoot. The performances of obtained controlled structures in actuating disk drive and their results are analyzed in later sections.

III. FRACTIONAL ORDER CONTROL

A. Fractional Order Controllers: A brief Introduction

The idea of a FOPID or $PI^{\lambda}D^{\mu}$ controller derives its origin from the concept of fractional order differentiation and integration [10]. Though popular definitions of fractional derivative like Grunwald-Letnikov and Riemann Loville definitions are prevalent, in terms of fractional order systems Caputo definition is widely preferred [11]. This definition of fractional derivative is used to derive fractional order transfer function models from fractional order differential equations with zero initial conditions. According to Caputo's definition the α^{th} order derivative of a function *f(t)* with respect to time is given by following equation.

$$D^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{D^m f(t)}{(t-\tau)^{\alpha+1-m}} d\tau$$

$$\alpha \in \mathbb{R}^+,\ m \in \mathbb{Z}^+,\ m-1 \le \alpha < m, \qquad (3.1)$$

Laplace transformation of Caputo's derivative results in *"s"* domain representation of Eq (3.1) and is provided in Eq (3.2)

$$\int_0^\infty e^{-st} D^\alpha f(t) dt = s^\alpha F(s) - \sum_{k=0}^{m-1} s^{\alpha-k-1} D^k f(0) \qquad (3.2)$$

where $\Gamma(\alpha) = \int_0^t e^{-t} t^{\alpha-1} dt$ is the Gamma function

$F(s) = \int_0^\infty e^{-st} f(t) dt$ is Laplace transform of $f(t)$

With an assumption of zero initial conditions the time domain operator D^α can be simply represented in frequency domain as s^α. A negative sign in the order of derivative ($-\alpha$) indicates a fractional integral operation. Hence the FOPID controller is a sum of fractional operators along with controller gains. The transfer function of a FOPID controller is given in Eq (2.7) and is represented once again in Eq (3.3)

$$C(s) = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu \qquad (3.3)$$

This typical controller consists of three controller gains $\{K_p, K_i, K_d\}$ and two more fractional order operators $\{\lambda,\mu\}$. For Instance, if $\lambda=1$ and $\mu=1$ Equation (3.3) reduces to classical controller in parallel structure. In order to implement a controller of form Equation (3.3) Oustaloup's band limited frequency domain rational approximation technique is used in the present paper and also in most of FO control literatures [12].

B. Digital Realization of Fractional Orders

The rationale behind the choice of frequency domain rational approximation of FOPID controller is that it can be easily implemented in real hardware using higher order analog or digital filters, corresponding to each fractional order differentiation or integration in FOPID controller.

The infinite dimensional nature of fractional order differentiator and integrator in FOPID controller structure creates hardware implementation issues in industrial application of FOPID controllers. However, recent research results demonstrated that band-limited implementation of FOPID controllers using higher order rational transfer function approximation of the integro-differential operators give satisfactory performance in industrial applications [13]. Oustaloup's recursive approximation [14], which has been implemented to realize fractional integro-differential operators in frequency domain, is given by the following Eqn's.

$$s^\alpha = K \prod_{k=-N}^{N} \frac{s+\omega_k'}{s+\omega_k} \qquad (3.4)$$

Here the poles, zeros and gain of the filter can be recursively evaluated as:

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+n+\frac{1}{2}(1+\alpha)}{2N+1}} \quad (3.5)$$

$$\omega_k^{'} = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+n+\frac{1}{2}(1-\alpha)}{2N+1}} \quad (3.6)$$

where $K = \omega_h^q$. In above equation set α is the order of the differ-integration, *(2N+1)* is the order of the filter and $(\omega_b,\omega_h)$is the expected fitting range. In the current study, 5th order Oustaloup's recursive approximation is done for the integro-differential operators within a frequency band of the constant phase elements (CPEs) as $\omega \in \{10^{-3}, 10^{3}\}$ rad/sec.

IV. Differential Search Algorithm

Differential Search (DS) algorithm is a new swarm intelligence algorithm proposed by Civicioglu [7] for solving optimization problems. Unlike state-of-art optimizers like PSO, ABC, Cuckoo Search and BFO etc., the search process of DS algorithm is unique and it mimics the *Brownian-like random-walk movement*. The biological motivation of DS algorithm reflects migration behavior of living beings ,which move away from a habitat where capacity and efficiency of food resources are depleted. The migration event involves movement of large number of individuals constituting a *superorganism.* A *superorganism* wanders and changes its current position towards more fruitful areas. Once a *superorganism* explores new fertile area (referred to as *stopover* site) it settles in the new site at least for a time and continues its migration towards more fertile areas. As said earlier, movement of *superorganism* can be ascribed to a Brownian-*like random-walk model* [7]. DS algorithm initiates by generating individuals of random solutions (of respective optimization problem) corresponding to a migrating artificial-*superorganism.* Henceforth artificial-*superorganism* tries to migrate from its position to global minimum value.

In DS algorithm each individual (or artificial-organisms) of a *Superorganism* is represented as $X_i, i = \{1,2,3,...,N\}$ and contains as many members as the size of problem i.e., $x_{ij}, j = \{1,2,3,...,D\}$. Here *N* signifies number of elements (total number of individuals) in the *superorganism* and *D* indicates size of the respective problem. The step by step procedure of DS algorithm is provided below

[**Step 1**] Each member of an artificial-organism is initialized to a random position using Equation 4.1.

$$x_{ij} = rand \times (up_j - lb_j) + lb_j \quad (4.1)$$

where *up* and *lb* corresponds to upper and lower bounds of dimension *j* of respective problem. X_i indicates an individual in an artificial-superorganism, $Superorganism_g$ = $[X_i]$ represents total number of individuals in g_{th} generation (*g=1, 2, 3…maxgeneration*).

[**Step 2**] p_1 and p_2 values [7] are used to determine frequency of perturbation of members in a position corresponding to an individual. *Scale* value is used to determine amount of perturbation in size of position of the members in an individual. *Scale* value is produced by using a gamma random number generator. The methodology used for calculation of *scale* value can be found in [7].

[**Step 3**] *Brownian-like random walk model* is employed to determine intermediate *stopover* sites in migration event. Randomly selected individuals move towards targets of *donor* =$[X_{random\_shuffling(i)}]$ in order to explore *stopover* sites. A *stopover* site position is produced using Equation 4.2

$$StopoverSite = Superorganism + Scale \times (donor - Supeorganism) \quad (4.2)$$

[**Step 4**] If a *stopover* site is more fertile than the sources associated with an individual of artificial-organisms, the corresponding individual moves to *stopover* site. The search for global minimum continues and the individual halted near intermediate *stopover* continue its search from current position.

V. Simulations And Results

A. Experimental Setup

The simulations are carried out for a time of 50ms, as the operation of computer applications should be in range of nano or micro seconds. Parameters involved in HDD are provided in Table 1. In DS algorithm p_1 and p_2 are set to 0.3 and no. of functional evaluations are 3000. A maximum of 25 individual trails are set for the algorithm to measure statistical quantities.

B. Results and Discussions

Figure 4 shows the step response of IAE (J_1) based DSA tuned PID/FOPID controller for head positioning servomechanism. It is clear from Figure that the FOPID controller tried to track the command signal i.e., tried to place the read head to desired track without any oscillations or overshoots. On other hand though PID controller tried to place the reader head with in less time it provided undesirable overshoots with also high steady state error, which can be observed from Table 2. To get even better controller parameters we also considered objective functions including time factor i.e., ITAE & ITSE. Figure 5 shows the time response of ITSE based PID/FOPID controller performance. There was slight improvement in settling time when compared to IAE based performance with reference to FOPID controller and remaining indices values are deteriorated.

Hence we chose ITAE or J_3 as objective function and tuned the controllers for optimum values. Fig 6 depicts the optimal responses obtained via ITAE. Both PID and FOPID gave better performances when compared to former objective functions. While FOPID in this case outperformed remaining methods, it gave a smooth response with less steady state error. Further to see the frequency domain performance of proposed method we obtained Bode plots. Figures (8-10) shows various plots for objective functions considered. From all these figures it is evident that due presence of fractional operators FOPID controllers provided flat phases with good margins (Table 2). On other hand PID controllers lead the system to instability.

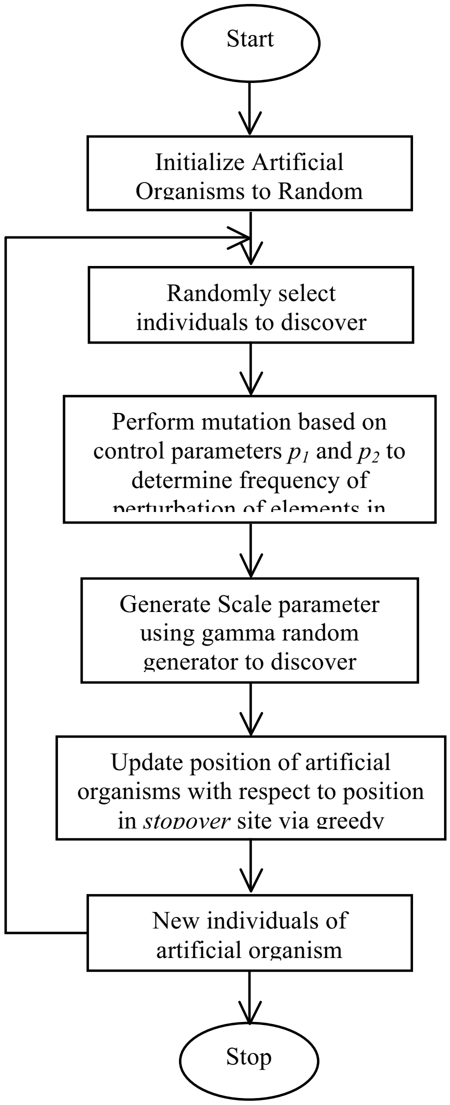

Figure 4. Differential Search (DS) Algorithm

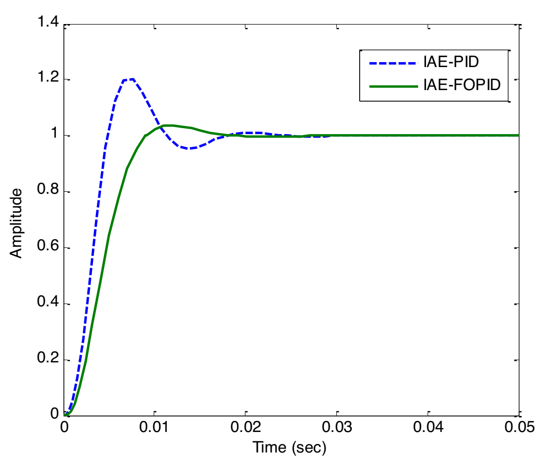

Figure 5. Step response of IAE based DSA tuned PID/FOPID Controller

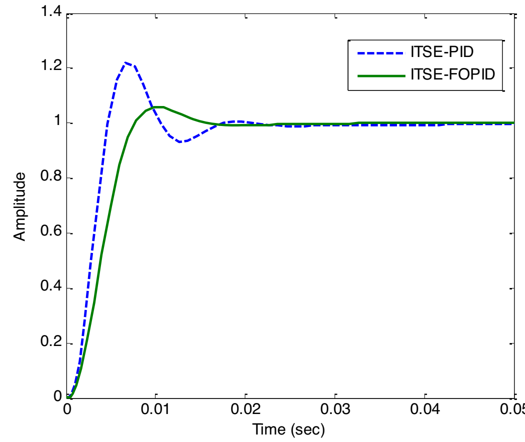

Figure 6. Step response of ITSE based DSA tuned PID/FOPID Controller

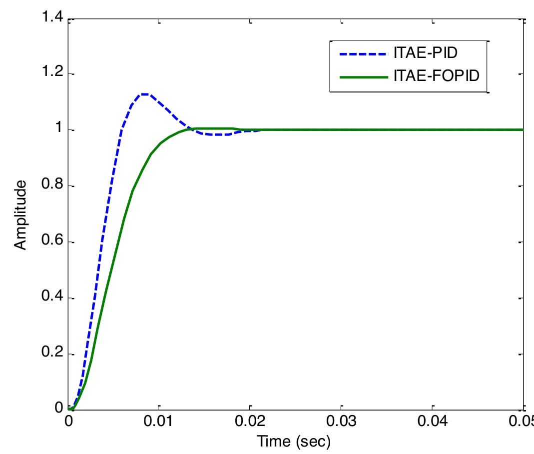

Figure 7. Step response of ITAE based DSA tuned PID/FOPID Controller

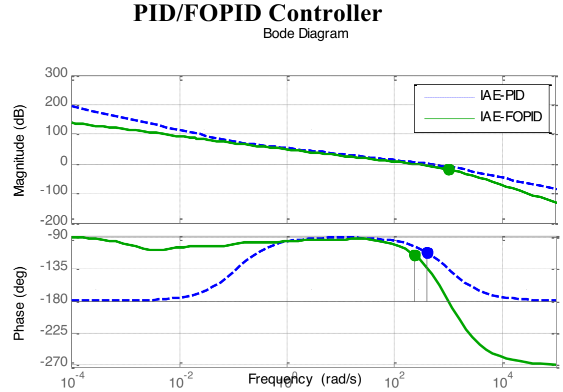

Figure 8. Bode plot of DSA tuned IAE-PID/FOPID based head positioning servo mechanism

Table 2. Optimal controller values and their time & frequency domain indices

Control	*Mean (std)*	K_P	K_i	K_d	λ	μ	*Po (%)*	*Risetime* (t_r)	*Settling time* (t_s)	*Steady Err*	*Gain Margin*	*Phase Margin*
						J_1=IAE						
DSA-PID	1.3811e-02 (1.4871e-07)	1757.9	200.00	87.111	-	-	20.176	**0.0046**	0.0166	7.903e-02	-Inf	68deg
DSA-FOPID	**3.9581e-03 (5.3084e-07)**	889.54	200.00	47.22	0.3484	1.0000	**3.6105**	0.0052	**0.0146**	**3.276e-03**	**18.4 dB**	**64.5 deg**
						J_2=ITSE						
DSA-PID	5.3368e-05 (2.2126e-12)	1436.2	200.00	94.99	-	-	22.118	**0.0046**	0.0196	5.022e-01	-Inf	67.1deg
DSA-FOPID	**4.02193e-06 (5.6081e-17)**	761.87	200.00	54.497	0.29508	0.9592	**5.9565**	0.0059	**0.0130**	**1.978e-01**	**17.3 dB**	**62.2 deg**
						J_3=ITAE						
DSA-PID	1.4636e-04 (3.7483e-08)	1384.9	91.324	68.172			12.585	**0.0058**	0.0176	8.483e-02	-Inf	72 deg
DSA-FOPID	**1.3521e-05 (3.7483e-10)**	691.4	200.00	38.903	0.25436	0.9890	**0.6803**	0.0070	**0.0112**	**1.089e-03**	**20.1 dB**	**68.8 deg**

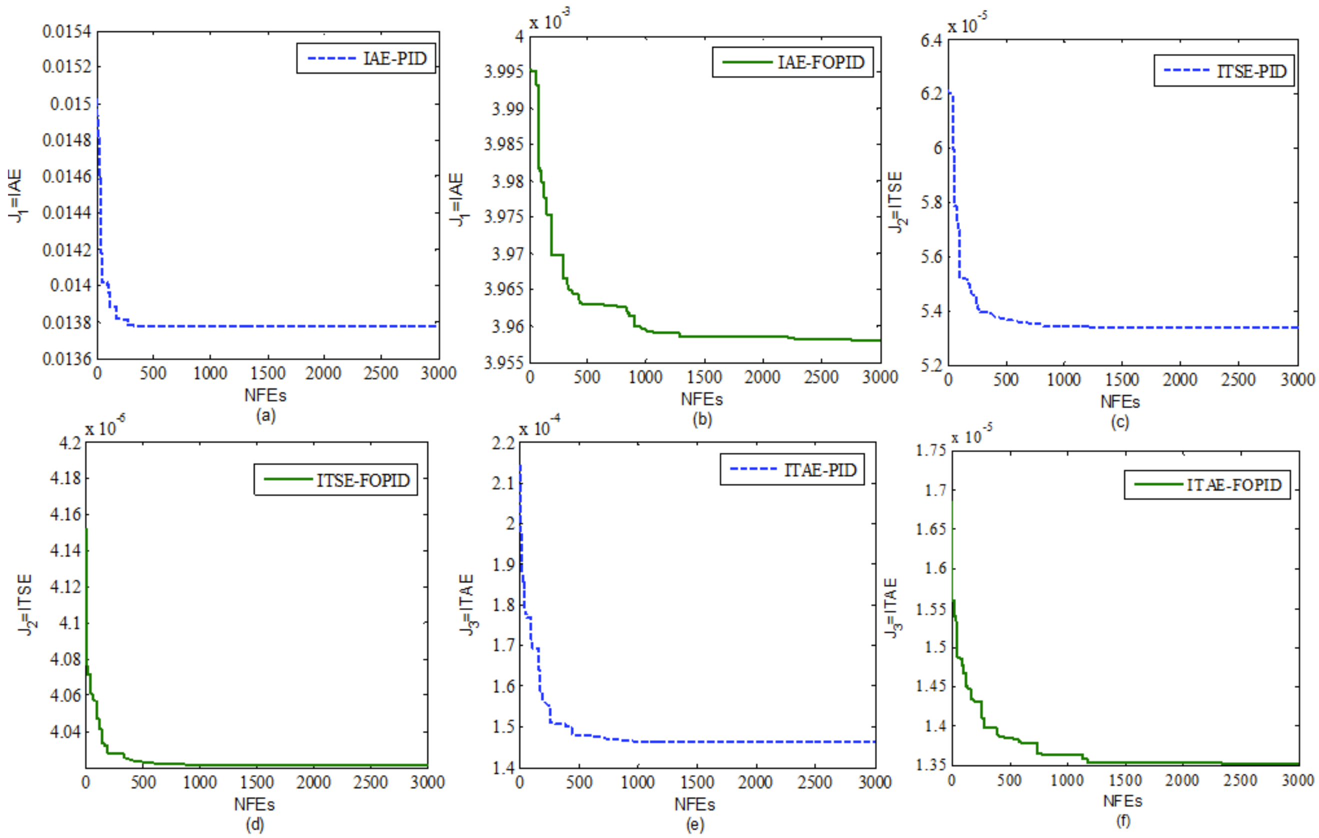

Figure 11. Convergence of DS algorithm towards minimum for various objective functions considered.

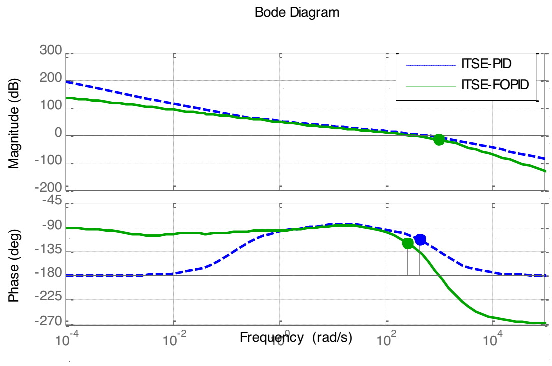

Figure 9. Bode plot of DSA tuned ITSE-PID/FOPID based head positioning servo mechanism

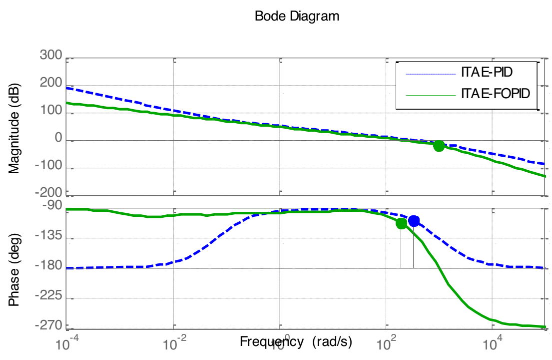

Figure 10. Bode plot of DSA tuned ITAE-PID/FOPID based head positioning servo mechanism

Convergence graphs of DS algorithm towards minimum are provided in Figures 11 (a-f) and corresponding statistical values are provided in Table 2.

VI. CONCLUSIONS

This paper deals with the application of intelligently tuned Fractional order PID controller for head positioning servomechanism in a Hard Disk Drive. A new swarm intelligence based Differential Search (DS) algorithm is used tune the controllers for optimal parameters. Suitable objective functions based on time domain optimality criterion are being considered and the results are analyzed in both time and frequency domain. From the simulations and results section it is clear that FOPID controllers gave optimal head positioning when compared to PID controllers.

Our future scope would focus on designing multi-objective frame work for designing fractional order controller in a disk drive by considering the external disturbances as well.

ACKNOWLEDGEMENTS

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REFERENCES

[1] K.C. Tan, R. Sathikannan, W.W. Tan and A.P. Loh, "Evolutionary design and implementation of hard disk drive servo control system," Softcomputing **11**(2), pp. 131-139, 2007.

[2] H. Hanselmann and A. Engelke, "*LQG*-control of a higly resonant disk drive head positioning actuator," *IEEE Trans Ind Elec.,* **35**(1), pp. 100-104.

[3] S. Weerasooriya and T. Phan, "Discrete-time LQG/LTR design and modeling of a disk drive actuator tracking servo system," *IEEE Trans Ind Elec*, **42**(3), pp. 240-247, 1995.

[4] TB. Goh, Z. Li, BM. Chen, TH. Lee and T. Huang, "Design and implementation of a hrad disk drive servo system using robust and perfect tracking approach. *IEEE Trans Contr Syst Tech*, **9**(2), pp. 221-233, 2001.

[5] M. Hirata, T. Atsumi, A. Murase, K. Nonami, "Following control of a hard disk drive by using sampled-data H_∞ control," In: Proc *IEEE Int. Conf Control App, Hawaii, USA,* pp. 182-186.

[6] I. Pan and S. Das, "Intelligent Fractional Order Systems and Control," *Studies in Computational Intilligence,* Springer Press, 2013.

[7] P. Civicioglu, "Transformin Geocentric Cartesian Coordinates to Geodetic Coordinates by Using Differential Search Algorithm", *Computers and Geosciecnes,* **46**, pp. 229-247, 2012.

[8] T.B. Reddy and M.J. Nigam, "Design and Implementation of a Hard Disk Drive Read/Write Head Controller Using FPGA for Optimal Performance," *Journal of Physcial Sciences,* vol. 11, pp. 185-198, 2007.

[9] R.C. Dorf and R.H. Bishop, "Modern Control Systems," Addison-Wesley, Eight ed, 1999.

[10] K.B. Oldham and J. Spanier, The Fractional Calculus, Academic Press, San Diego, 1974.

[11] S. Das, Functional Fractional Calculus, Springer, 2011.

[12] D. Valerio and Sa da Costa, "Introduction to Single-input, Single-output Fractional Control," *IET Control Theory Appl* **5**(8), pp. 1033-1057, 2011.

[13] M.O. Effe, "Fractional order systems in industrial automation-a survey" *IEEE Tran Ind Inform* **7**(4), pp. 582-591, 2011.

[14] A. Oustaloup, F. Levron, B. Mathiu, F.M Nanot, "Frequency-band complex noninteger differentiator: characterization and synthesis," *IEEE Transactions on Circuits and Systems I: fudamental Theory and Applications, I,* **47**(1), pp. 25-39, 2000.